The Geometric Lock: Mastering the Sudoku X-Wing Strategy

Research Report

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1 Introduction: Beyond the Box

In the progression of Sudoku solving techniques, the **X-Wing** represents a significant leap. While basic strategies like "Scanning" and "Locked Candidates" (Pointing/Claiming) rely on the intersection of a block and a line, the X-Wing abandons the concept of the 3×3 block entirely. It is a "Single Digit Pattern" technique that relies exclusively on the geometric relationship between rows and columns.

The X-Wing is the fundamental "Fish" pattern (Size 2 Fish). It exploits a state of **parallel uncertainty**: we may not know exactly *where* a number goes, but we know it must exist in a rectangular formation that forces eliminations elsewhere.

2 The Theoretical Framework

2.1 The Logic of Conjugate Pairs

An X-Wing occurs when a specific candidate digit (let's call it k) is restricted to exactly **two** positions in two specific lines (the "Base Sets"). If these two positions align perfectly across the grid, they form the corners of a rectangle.

The Theorem:
If digit k appears in Row R_1 only in columns C_A and C_B, AND digit k appears in Row R_2 only in columns C_A and C_B:
Then digit k **must** be placed at the corners of the rectangle formed by intersections $(R_1, C_A), (R_1, C_B), (R_2, C_A), (R_2$

Since the solution for R_1 must be either C_A or C_B (and R_2 takes the opposite), the digit k is effectively "locked" into these two columns for the duration of these two rows.

Result: We can eliminate digit k from *all other rows* passing through columns C_A and C_B.

2.2 Taxonomy

- **Base Sets:** The two lines (rows or columns) where the candidate is restricted to only two cells.
- **Cover Sets:** The perpendicular lines (columns or rows) where the elimination takes place.
- **Horizontal X-Wing:** Base sets are Rows; eliminations happen in Columns.
- **Vertical X-Wing:** Base sets are Columns; eliminations happen in Rows.

3 Visual Analysis: Horizontal X-Wing (Row-Based)

In this scenario, we look for two rows where a candidate appears **only twice**, and those appearances share the same columns.

Scenario: Candidate **7**. **Base Sets:** Row 3 and Row 7.

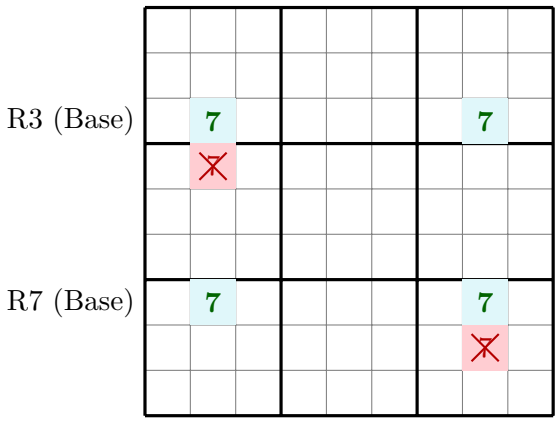

Figure 1: Horizontal X-Wing. Rows 3 and 7 (Base) claim the 7s for Columns 2 and 8.

The Logic Trace:

1. **Row 3 Analysis:** The 7 must be in either R3C2 or R3C8.
2. **Row 7 Analysis:** The 7 must be in either R7C2 or R7C8.
3. **Conclusion:** In either universe, there is a 7 in Column 2 (at row 3 or 7) and a 7 in Column 8 (at row 7 or 3).
4. **Elimination:** No other cell in Column 2 or Column 8 can contain a 7.

4 Visual Analysis: Vertical X-Wing (Column-Based)

This is the grid rotation of the previous example. We look for two columns where a candidate appears **only twice**.

Scenario: Candidate **4**. **Base Sets:** Column 4 and Column 6.

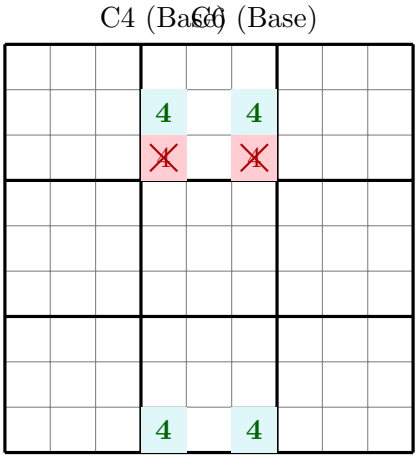

Figure 2: Vertical X-Wing. Columns 4 and 6 (Base) claim the 4s for Rows 2 and 9.

The Logic Trace:

1. **Scan Columns:** You notice that in **Col 4**, the digit 4 only appears in Row 2 and Row 9.
2. **Scan Columns:** You notice that in **Col 6**, the digit 4 *also* only appears in Row 2 and Row 9.
3. **Elimination:** Since columns 4 and 6 claim the 4s for Row 2 and Row 9, no other cell in **Row 2** or **Row 9** can contain a 4.

5 Step-by-Step Detection Guide

Spotting an X-Wing is significantly harder than spotting Pointing Pairs because the relationship spans the whole grid.

1. **Candidate Filtering:** This technique is nearly impossible to spot without candidate highlighting. Focus on one digit at a time.
2. **The "Two-by-Two" Scan:** Scan rows one by one. Stop at any row that has **exactly two**

instances of the target digit. Scan down the board for another row that has the target digit in **exactly those same two columns**.

3. **Verify the Base:** Ensure the restriction is in the *Base Lines* (e.g., the Rows). It does **not** matter if the columns contain other candidates for that digit.
4. **Execute:** Remove candidates from the Cover Sets (the perpendicular lines).

6 Mathematical Context: The "Fish" Family

For the advanced solver, understanding where the X-Wing sits in the hierarchy helps in spotting more complex patterns.

Pattern	**Size**	**Logic**	**Geometry**
X-Wing	2	2 Rows $\rightarrow$ 2 Cols	Rectangle
Swordfish	3	3 Rows $\rightarrow$ 3 Cols	Grid/Mesh (3x3 corners)
Jellyfish	4	4 Rows $\rightarrow$ 4 Cols	Complex Grid

Table 1: The Fish Family Hierarchy

The X-Wing is technically a **Size 2 Fish**. The logic is always: "N candidates are restricted to N rows, residing in only N columns."

7 Conclusion

The X-Wing is the gateway to professional Sudoku solving. It teaches the player to stop looking at "boxes" and start seeing the grid as a matrix of intersecting coordinate lines. While they appear less frequently than basic Locked Candidates, their discovery often breaks "deadlocked" puzzles where no simple moves remain.