The Geometric Logic of Bivalue Triads: A Deep Dive into the Y-Wing Strategy

Sudoku Analytical Research

December 16, 2025

1 Introduction: The "Bent" Triple

In the hierarchy of Sudoku solving techniques, the **Y-Wing** (formally known as the **XY-Wing**) represents the first major step away from single-house pattern recognition into multi-house geometry. While intermediate solvers are comfortable with "Naked Triples" (three cells in a *single* row, column, or box sharing three candidates), the Y-Wing applies a similar logical pressure across *intersecting* houses.

Fundamentally, the Y-Wing is a specific configuration of three **bivalue cells** (cells containing exactly two candidates). It serves as a "forcing pattern" involving exactly three distinct digits. Because of its reliance on bivalue cells and "If-Then" logic, it is mathematically classified as the shortest possible **XY-Chain** (Length 3). However, due to its distinct geometric shape and frequent occurrence, it is universally treated as a standalone technique.

2 Anatomy of a Y-Wing

To identify a Y-Wing, the solver must locate a specific triad of cells that satisfies strict candidate and visibility constraints. Let us define the three candidates involved as **A**, **B**, and **C**.

2.1 The Three Components

1. **The Pivot (The Hinge):** The central cell. It must "see" both Pincers. It contains candidates **{A, B}**.
2. **Pincer 1 (Wing 1):** Shares a house with the Pivot. Contains candidates **{A, C}**. (Shares 'A' with Pivot).

3. **Pincer 2 (Wing 2):** Shares a *different* house with the Pivot. Contains candidates **{B, C}**. (Shares 'B' with Pivot).

2.2 Visualization: The "Bent" Shape

Unlike a Naked Triple, the Y-Wing forms a corner.

	Col 2	...	**Col 8**	...
Row 2	**{1,2} Pivot**	...	**{1,3} Pincer 1**	...
...	...	...	...	...
Row 9	**{2,3} Pincer 2**	...	**Target Cell**	...

Table 1: Diagram 1: Standard Y-Wing Configuration (C=3)

3 The Logical Proof

The Y-Wing works on a principle of **Inescapable Truth**. We do not know *which* Pincer contains the common candidate **C**, but we can prove that **one of them must**.

3.1 The "If-Then" Fork

Let us analyze the state of the **Pivot {A, B}**.

- **Reality 1:** If Pivot is **A** → Pincer 1 ({A,C}) cannot be A → **Pincer 1 is C**.
- **Reality 2:** If Pivot is **B** → Pincer 2 ({B,C}) cannot be B → **Pincer 2 is C**.

Since the Pivot must be A or B, **Candidate C must exist in either Pincer 1 or Pincer 2**.

4 The Elimination Logic (The Kill Zone)

Because we have proven that C exists in *at least one* of the Pincers, we can eliminate C from any cell that intersects **both** Pincers.

> **The Rule:** The Kill Zone is any cell that shares a house with Pincer 1 AND a house with Pincer 2. It does not need to see the Pivot.

In Diagram 1 above, if the cell at **Row 9, Column 8** contains a 3, it can be eliminated because it sees Pincer 1 (via Column 8) and Pincer 2 (via Row 9).

5 Common Pitfalls

1. **Pivot Blindness:** The Pivot *must* see both Pincers. If the middle cell doesn't see the others simultaneously, it is not a Y-Wing.
2. **Wrong Elimination:** Users often try to eliminate 'A' or 'B'. The elimination *only* applies to **Candidate C** (the one shared by the Pincers).
3. **Invalid Candidates:** Ensure the Pincers share the 'C', but the Pivot contains the other two ('A' and 'B').

6 Summary Table

Component	Candidates	Requirement
Pivot	{A, B}	Must see both Pincers.
Pincer 1	{A, C}	Must see Pivot. Shares 'A' with Pivot.
Pincer 2	{B, C}	Must see Pivot. Shares 'B' with Pivot.
Target	Contains C	Must see **both** Pincers.

Table 2: Y-Wing Rule Summary